

3.17 Graphs of Polar Equations

Polar Equations and Graphs

An equation whose variables are polar coordinates is called a **polar equation**. The **graph of a polar equation** consists of all points whose polar coordinates satisfy the equation.

Identify and graph a polar equation.

Rose Curves

$r = a \sin(n\theta)$ or $r = a \cos(n\theta)$, n is an integer greater than 1

Domain: All reals

Range: $[-|a|, |a|]$

Symmetry: n even, symmetric about x -axis, y -axis, and origin

n odd, $r = a \cos(n\theta)$ symmetric about x -axis

n odd, $r = a \sin(n\theta)$ symmetric about y -axis

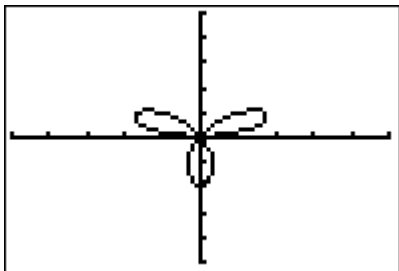
If n is odd there are n petals.

If n is even there are $2n$ petals.

Maximum r value: $|a|$

Example: (Also see page 251 in book.)

$$r = 2 \sin(3\theta)$$



Limacon Curves

$$r = a \pm b \sin \theta \quad \text{and} \quad r = a \pm b \cos \theta, \quad a > 0 \text{ and } b > 0$$

Domain: All reals

Range: $[a - b, a + b]$

Symmetry: $r = a \pm b \sin \theta$, symmetric about y -axis

$r = a \pm b \cos \theta$, symmetric about x -axis

Maximum r value: $a + b$

Limacon with an inner loop: $\frac{a}{b} < 1$

Cardioid: $\frac{a}{b} = 1$

Dimpled: $1 < \frac{a}{b} < 2$

Convex: $\frac{a}{b} \geq 2$

(See page 251 in book for examples of Limacon Curves)

Lemniscate curve: $r^2 = a^2 \sin 2\theta$ and $r^2 = a^2 \cos 2\theta$, $a > 0$

(See page 252 for example of Lemniscate Curves)

Spiral of Archimedes: $r = \theta$

Domain: All reals

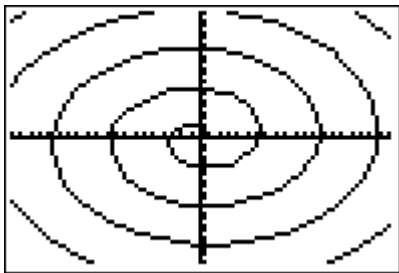
Range: All reals

Symmetric about the y-axis

No maximum r value

Example:

$$r = \theta$$



Lines

Description: Line passing through the pole making an angle θ with the polar axis.

Rectangular equation: $y = (\tan a) x$, $x = a$, $y = b$

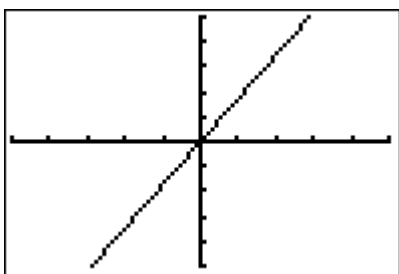
Polar Equations: $\theta = a$ (diagonal line through origin with angle a to the x -axis),

$r \cos \theta = a$ or $r = a / \cos \theta$ (vertical line),

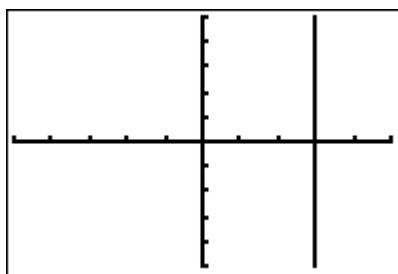
$r \sin \theta = b$ or $r = b / \sin \theta$ (horizontal line)

Examples:

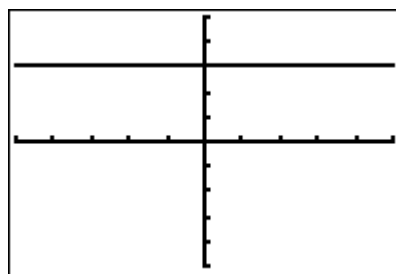
$$\theta = \pi/3$$



$$r = 3 / \cos \theta$$



$$r = 3 / \sin \theta$$



Circles

Description: Center at the pole, radius a

Rectangular equation: $a^2 = x^2 + y^2$, $a > 0$

Polar Equation: $r = a$, $a > 0$

Example:

$$r = 3$$

